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THESIS

A VIBRATING-REED MASS-FLOW-METER

by

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A Vibrating-Reed Mass-Flow-Meter

by

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ABSTRACT

For many fluid-mass-rate-of-flow metering situations, a measure of the fluid's density-velocity product is required. The density-velocity (pv) product is multiplied by an effective conduit cross-sectional area to yield the mass-rate-of-flow. The area multiplication is accomplished by simply changing the scale of the pv-product indicator.

The purpose of this paper is to show how a magnetically-driven vibrating reed can be used to measure either the pv product of a fluid or its mass-rate-of-flow through a conduit. The proposed meter differs from the rotating-vane mass-rate-of-flow meters in that it operates on a transverse rather than angular momentum exchange.

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LIST OF SYMBOLS

p	Density of fluid.
$\bar{v}$	Velocity of fluid.
$\bar{v}_p$	Velocity of reed.
$\bar{v}_r$	Relative wind.
v	Magnitude of fluid velocity.
v_p	Magnitude of reed velocity.
v_r	Magnitude of relative wind.
pv	Density flow rate (Product of p and v),
a	Angle of attack.
C_L	Coefficient of lift.
A	Area of reed.
V_{po}	Amplitude of sinusoidial reed velocity.
k	Constant of proportionality.
k'	Constant of proportionality.
K	Constant of proportionality.
b	Constant of proportionality.
i_d	Magnetic-driver coil current.
f	Frequency.
w	Radian frequency.
V	Velocity.
h	Height of fluid layer.
W	Width of reed.
L	Length of reed.
F	Lift force on reed.
m	Mass
B	Alignment error angle.
t	Time.
V_g	Signal generator voltage.
kp	Slope of linear portion of curve.
V_{gL}^2	Square of voltage required to overcome mechanical losses.
C_D	Complex drag coefficient.
T	Time interval.

ACKNOWLEDGEMENT

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I. INTRODUCTION

The purpose of this paper is to propose a new type of fluid-mass-rate-of-flow meter. The device proposed is basically a vibrating reed mounted in a fluid stream. A magnetic driver is used to produce the vibrations. The fluid's pv product (or mass-rate-of-flow indication) can be read directly with no external analog or digital computations necessary. The device is readily adaptable to many mass-rate-of-flow metering situations. Moreover, the meter can be made fairly rugged. Since there is only one moving part (the reed) and no pivots or bearings, the device should hold up well in a corrosive enviroment (assuming that the exposed metal parts are properly coated).

Section II contains the development of theory for two basic approaches to obtain a pv-product indication from a vibrating reed. The first approach assumes constant amplitude and frequency with the input current to the magnetic driver as the pv indication. The second approach assumes constant input current to the driver and the amplitude of reed vibrations serves as the pv indication.

Section III gives a description of a crude model constructed by the author to demonstrate the validity of the theory in section II.

Section IV presents some rudimentary wind-tunnel data collected on the model.

In section V, basic conclusions are drawn and some practical suggestions are offered on how to construct a practical working meter.

No attempt was made to investigate in detail such secondary effects as wind-tunnel blockage, aeroelasticity effects, mechanical and magnetic losses. The specific purpose of this paper is simply to introduce the theory of operation and to demonstrate that a working model can be made. At no point in the investigation were any state-of-the-art techniques employed.

II. THEORY OF OPERATION

In this section, the method and basis for using a vibrating reed to obtain a measure of a moving fluid's pv-product are developed.

A. CONSTANT REED-AMPLITUDE APPROACH

Assume that a flat reed is oriented in a moving fluid of velocity $\bar{v}$ as depicted in figure 1. The reed is driven with a velocity $\bar{v}_p$ which is perpendicular to the fluid velocity.

Top View   Side View

Figure 1. Orientation Of Reed In Fluid Stream.

The relative "wind" seen by the reed is denoted as $\bar{v}_r$ and is equal to $\bar{v} - \bar{v}_p$. If the restriction $v > 3v_p$ is imposed, then the force required to drive the reed in the direction of $\bar{v}_p$ is given by,

$$\text{Lift Force} = \tfrac{1}{2} \, C_L \, p \, v_r^2 \, A. \qquad (2.1)$$

where:

C_L = coefficient of lift

p = density of fluid

A = area of reed

The area A will be considered a constant in the following discussion.

Assuming small angles of attack, ie, $v > 3v_p$, the coefficient of lift for a symmetrically flat reed is given by

$$C_L = k \, \sin(a). \qquad (2.2)$$

Using the relationship from figure 1,

$$\sin(a) = \frac{v_p}{v_r} \qquad (2.3)$$

together with equation (2.2), we can replace C_L in (2.1) to obtain

$$\text{Lift Force} = \tfrac{1}{2} \, k \, \frac{v_p}{v_r} \, p \, v_r^2 \, A \qquad (2.4)$$

$$= k' \, v_p \, p \, v_r ,$$

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where k' denotes the constant $\frac{1}{2}kA$. For small values of $\frac{v_p}{v}$, the relative "wind" $\bar{v}_r$ has approximately the same magnitude as the fluid velocity $\bar{v}$ and equation (2.4) can be rewritten as

$$\text{Lift Force} = k' \, v_p \, (pv). \qquad (2.5)$$

The reed velocity has not been restricted to any constant value. If, for example, the reed velocity is a sinusoidal function of time, ie,

$$v_p(t) = V_{po}\sin(wt), \qquad (2.6)$$

then equation (2.5) becomes

$$\text{Lift Force} = k' \, (pv) \, V_{po}\sin(wt). \qquad (2.7)$$

Taking the average of both sides yields

$$\text{Average Lift Force} = \frac{2}{\pi} k' \, (pv) \, V_{po}$$
$$= K \, (pv) \, V_{po}. \qquad (2.8)$$

Equation (2.8) is the basis upon which the vibrating-reed pv (or mass-rate-of-flow) meter is to operate. If V_{po} is held constant and pv is allowed to vary, equation (2.8) states that the <u>average lift force will be directly proportional to the pv-product!</u>

To obtain a pv meter based on equation (2.8) it is necessary only to drive a reed at a constant amplitude and frequency and to measure the average lift force on the reed. An indication of the relative magnitude of the average lift force can easily be obtained, as outlined in the following paragraphs.

Consider an elastic metal reed mounted by one end onto a rigid support and vibrating at its natural frequency. The forces required for simple harmonic motion are supplied by internal stresses within the reed. If the mechanical losses of the reed and its mount are made sufficiently small, then the average lift force on the reed is approximately equal to the average external drive force that must be applied to sustain constant amplitude of oscillations. Assume that a magnetic driver is utilized for maintaining the reed oscillations. Then for a properly constructed driver, the force on the reed will be proportional to the square of the magnetic flux "seen" by the reed. Furthermore, for a magnetic driver possessing a soft iron core and operating in its linear region, the driver's magnetic flux is directly proportional to the driver's coil current. If hysteresis is neglected, the relationship of force to current is

$$\text{Magnetic Drive Force} = b \, i_d^2, \qquad (2.9)$$

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where the constant b denotes proportionality. This force is of course always oriente. in the same direction and tends to pull the reed toward the driver.

In particular, for a sinusoidal driver-coil current,

$$i_d = \sin\frac{w}{2}t \qquad (2.10)$$

equation (2.9) becomes

$$\text{Magnetic Drive Force} = b \left(\sin \frac{w}{2} t\right)^2$$

$$= b \left(\tfrac{1}{2} - \tfrac{1}{2} \cos wt\right). \qquad (2.11)$$

Thus, the magnetic force for a sinusoidal current consists of a d.c. term plus a sinusoidal term at double the frequency of the input current. So, to have the magnetic driver vibrate the reed at its natural frequency, the frequency of the coil current is made one-half that of the reed. Of course the reed should be rigid enough to prevent the d.c. component of the magnetic force from producing any substantial displacement of the average reed position.

The nature of the pv meter is now obvious! A reed is mounted onto a rigid base and a magnetic driver is used to maintain oscillations of a constant amplitude at the reed's natural frequency. The driver current will be a measure of the fluid's pv-product. In particular, the relationship of i_d^2 to the pv-product should be linear over the region in which the assumption $v > 3v_p$ is valid.

The author constructed a crude pv indicator along the lines suggested in the above paragraph. A description of the device together with some wind-tunnel data obtained on it are presented in sections III and IV.

B. CONSTANT DRIVER-CURRENT APPROACH

The restriction to constant reed-vibration amplitude, which was assumed in the previous approach, would obviously prove impractical for may situations in which a pv-product is desires. An alternate, simpler approach is also implied by equation (2.8). Consider equation (2.8) rewritten as

$$V_{po} = \frac{\text{Average Lift Force}}{K\ (pv)}. \qquad (2.12)$$

It has already been noted that the average drive force required to sustain oscillations is approximately equal to the average lift force - for sufficiently low val.:s of mechanical losses. For low losses, then,

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equation (2.12) can be modified to yield

$$v_{po} \simeq \frac{\text{Average Drive Force}}{K\ (pv)}. \qquad (2.13)$$

But the amplitude of oscillation can be expressed as

$$\text{Amplitude} = \frac{V_{po}}{w}, \qquad (2.14)$$

where w is the angular frequency of oscillation. Substitution of
equation (2.14) into (2.13) yields:

$$\text{Amplitude} \simeq \frac{\text{Average Drive Force}}{w\ K\ (pv)} \qquad (2.15)$$

Equation (2.15) states that with the average drive force (or driver cur-
rent) held constant, the amplitude of reed oscillations is approximately
inversely proportional to pv. Thus a measure of the amplitude is a
measure of the pv-product.

Figure 7 is a typical plot of reed-oscillation amplitude vs. wind
velocity (constant driver input current) for the author's crude model.

C. CONSERVATION OF LINEAR MOMENTUM

An alternate viewpoint to the operation of the meter is revealed by
applying the conservation of linear momentum principle. The reed is
viewed as continually transferring momentum to the fluid particles pas-
sing over its surface. The number of particles to which the reed
imparts a perpendicular component of momentum is readily seen to be
approximately proportional to the density of particles and to the
velocity of the fluid. Using the concept of linear momentum, equation
(2.5) can be deduced by considering the reed's effect on a thin layer of
fluid of height h passing immediately above and in contact with the reed.
Laminar flow and a value of $v \gg v_p$ are assumed.

Start with the formula equating impulse to change of momentum.

$$\int_{t_1}^{t_2} F\ dt = \wedge(mv) \qquad (2.16)$$

Assuming a steady-state velocity $\bar{v}_p$ for the reed and noting that each
fluid particle in the thin layer undergoes a net change in velocity of
v_p, one can rewrite equation (2.16) as

$$F\ (t_2 - t_1) = v_p \triangle m. \qquad (2.17)$$

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The quantity Δm is the average fluid mass undergoing a change in velocity of v_p during the time interval $t_2 - t_1$ or,

$$\Delta m = p \ \Delta \text{Volume}$$
$$= p \ h \ W \ L \ v \ (t_2 - t_1) \tag{2.18}$$

where h, W, and L are the linear dimensions of the layer and v is the velocity of the fluid impinging on the reed. Substituting equation (2.18) into equation (2.17) and simplifying yields

$$F = h \ W \ L \ v_p \ (pv), \tag{2.19}$$

which is identical to equation (2.5) with k' replaced by h W L.

The extension of equation (2.19) to include all other layers lying above and below the reed requires the application of a partial differntial equation in three dimensions - a difficult but perfectly legitimate approach. A rigorous derivation based on linear momentum will not be attempted in this paper since the previous development based on the empirical lift equation yields a simpler description of the meter's operation.

D. EFFECT OF A SMALL ALIGNMENT ERROR

The derivations in the earlier portions of this section were based on the assumption that the direction of the fluid flow was tangential to the reed's surface. We will now consider the effect of a small alignment error.

Assume the reed to be inclined a small angle B to the fluid velocity vector $\bar{v}$ as pictured in figure 2 below.
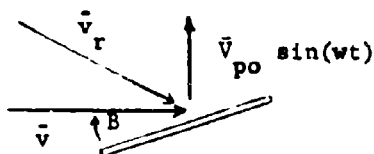

FIGURE 2.   Orientation Of Reed With Small Alignment Error.

For a symmetrically flat reed the lift equation is

$$\text{Lift Force} = \frac{1}{2} k \ \sin(a) \ p \ v_r^2 \ A. \tag{2.20}$$

The value of $\sin(a)$ above can be, for a given small alignment error B,

closely approximated by

$$\sin(a) = \frac{V_{po} \sin(wt)}{v_r} + B. \qquad (2.21)$$

Substitution of equation (2.21) into equation (2.20) yields:

$$\text{Lift Force} = \frac{1}{2} k \left[\frac{V_{po} \sin(wt)}{v_r} + B \right] p v_r^2 A$$

$$\triangleq \frac{1}{2} k p v_r V_{po} \sin(wt) A + \frac{1}{2} k B p v_r^2 A \qquad (2.22)$$

If, as before, it is assumed that the area A is essentially a constant for small angles of attack and that $v \approx v_r$, then equation (2.22) becomes

$$\text{Lift Force} \approx k' (pv) V_{po} \sin(wt) + k' B p v^2. \qquad (2.23)$$

For steady-state values of p and v, the term on the right is a constant. Its effect is to produce a constant displacement of the average position of the reed. However, since the reed has been made stiff enough to not be appreciably deflected by the d.c. component of the magnetic driving force, the constant lift force produced by the small alignment error B will also produce little deflection unless v becomes excessively high. Thus it can be concluded that a small alignment error will produce little effect on the amplitude of reed oscillations.

Figure 8 is a plot graphically demonstrating the effect of an intentional alignment error applied to the author's laboratory model. The plot lends support to the conclusion that the reed alignment is not an extremely critical adjustment. As can be seen from figure 8, an alignment error of three or four degrees produced no significant effect on the reed amplitude for a mid-range value of v.

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III. DESCRIPTION OF EQUIPMENT

To test the conclusions of the THEORY OF OPERATION section, the
author constructed a magnetically-driven vibrating-reed device along
the lines suggested in subsection II.A. Figure 3 is a pictorial des-
cription of the basic device. A dual-reed tuning fork arrangement was
chosen in order to reduce the mechanical losses - subsection V.A con-
tains a discussion of mechanical losses. The high-carbon steel reeds
were 2.5 inches long by 0.5 inches wide and vibrated with a natural
frequency of 67.4 Hz. The magnetic-driver coil was constructed of
approximately 400 turns of #29 copper wire wound onto a $\frac{3''}{8}$ soft-iron
core, and had a measured impedence of $7.6 + j2.2$ ohms at the drive fre-
quency of 33.7 Hz. Magnetic coupling of the reed to the driver was
enhanced by small iron bits mounted onto the reeds. The device was
tested in a small three-inch by three-inch square bench-model wind
tunnel located about 50 feet above sea level. Wind velocity could be
varied from 0 to 28 ft/sec and was measured with a pitot tube connected
to a calibrated inches-of-water scale.

Figure 4 is a block diagram dipicting the method used to check the
constant-reed-amplitude approach described in subsection II.A. An
aural detection scheme was used to hold the reed oscillation amplitude
constant. The driver current was adjusted to a point just below that
at which the reeds began to contact the pole pieces. Contact with the
pole pieces produced a sharp pinging sound. This method proved to be
more accurate and reliable than some optical schemes that were tried.

Figure 5 is a block diagram of the laboratory setup used to test
the constant driver-current approach of subsection II.B. In this
approach, the driver-current amplitude was held constant and the reed-
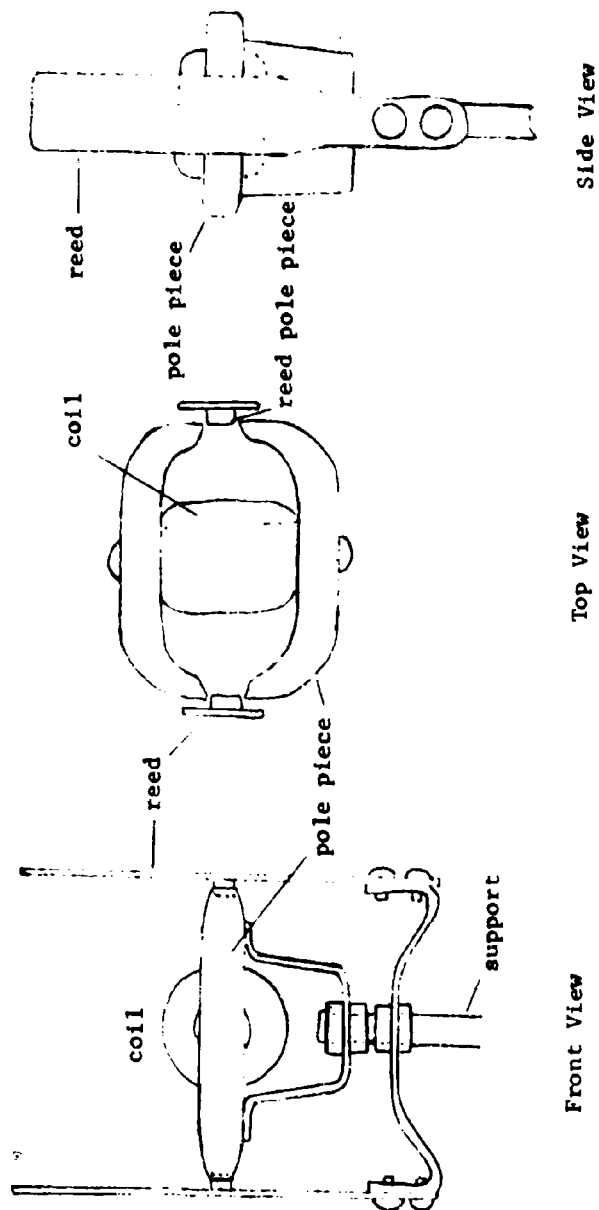oscillation amplitude was detected by a strain gage bonded to the reed's
base.
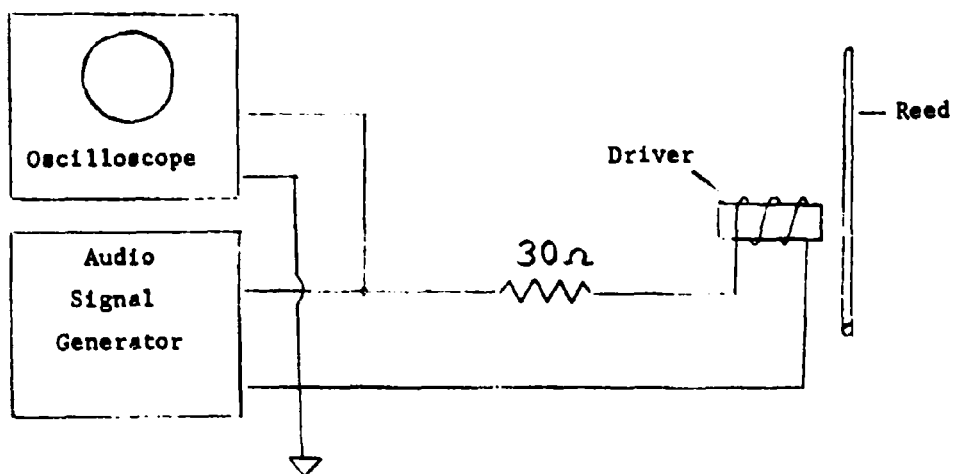
FIGURE 3. Magnetically Driven Dual-Reed Device

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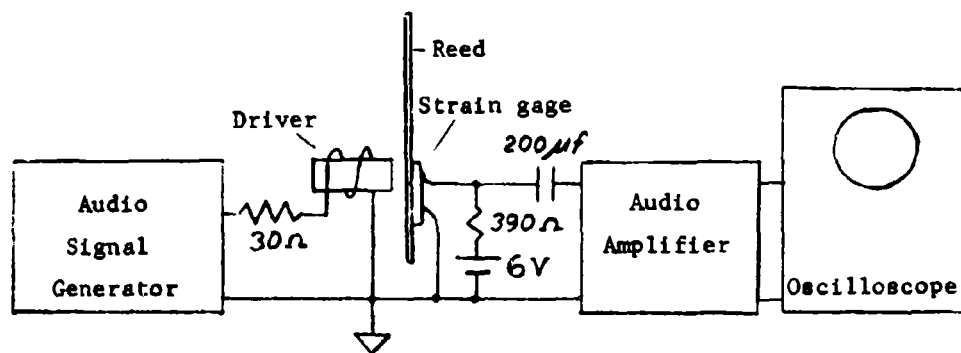FIGURE 4.   Constant Reed-Amplitude Approach

FIGURE 5.   Constant Driver-Coil-Current Approach

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IV. PRESENTATION OF WIND-TUNNEL DATA

Figure 6 shows two typical plots obtained with the constant-reed-amplitude-approach configuration of figure 4. The lower plot is for the dual-reed device of figure 3. The upper curve is for a single reed vibrating at a slightly larger amplitude. It was obtained by removing one of the two reeds from the device. The square of the generator voltage at point A is an indication of the driver current required to overcome the mechanical losses of the dual-reed device, while C minus A represents the additional power required to drag the reeds through still air. Points B and D represent the same quantities for the single-reed plot.

Comparison of the upper single-reed plot with the lower dual-reed plot offers considerable insight into the operation of the device. First, the mechanical losses are increased for the single reed, partly due to the unbalancing of the "tuning fork", and partly because the amplitude of vibrations has been increased. Secondly, the power required to drag the reed through still air has increased because of the increased amplitude of vibrations. The drag force is proportional to the square of the reed oscillation amplitude as explained later in subsection J of section V. Thirdly, the slope of the single-reed plot is greater, again because of the increase in oscillation amplitude. This increase in slope was correctly predicted by equation (2.8). Finally, the knee of the single-reed plot has been pushed to the right. This is because the value of v in the restriction $v > 3V_{po}$ has increased due to an increase in V_{po} (same frequency but larger amplitude). For a reed oscillation amplitude of one-eight inch, that value of v calculated for the reed tip is actually 13 ft/sec, so it appears that the restriction could be relaxed about 35%. In summary, an increased reed oscillation amplitude results in greater mechanical losses, a steeper slope, and the extension of the non-linear portion of the curve.

Considering the reed operation from a conservation-of-energy viewpoint, one would expect the slope of the single-reed plot to be less than that of the dual-reed plot, since less energy is required

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Square Of Generator Voltage (Peak to Peak)

vs

Wind Velocity (single reed device)

Generator Frequency = 33.7 Hz

Reed Frequency = 67.4 Hz
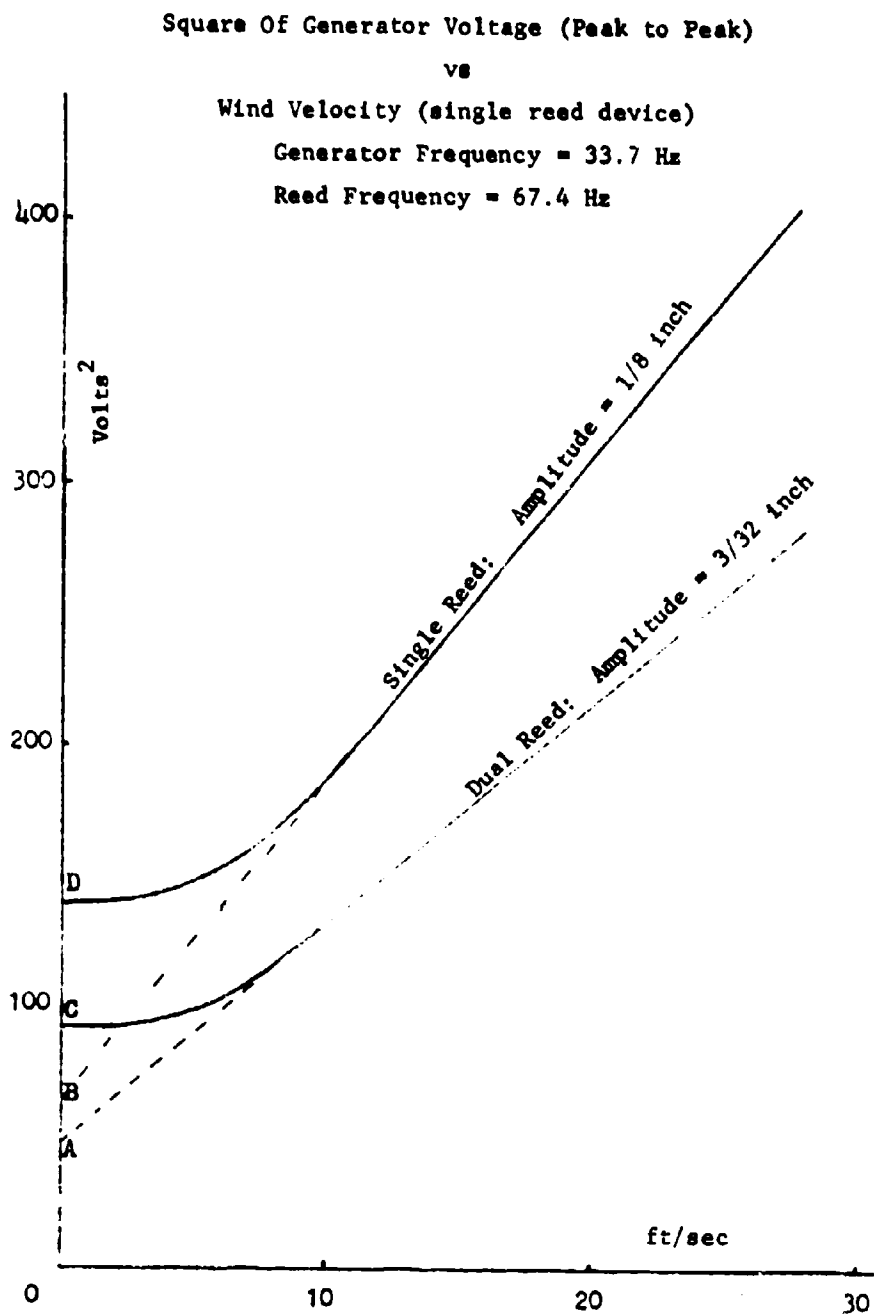
FIGURE 6. Constant-Reed-Amplitude Plots

LOG AMPLIFIED STRAIN-GAGE VOLTAGE
vs.
LOG WIND VELOCITY
for a
Single-Reed Device

Reed Frequency = 69 Hz
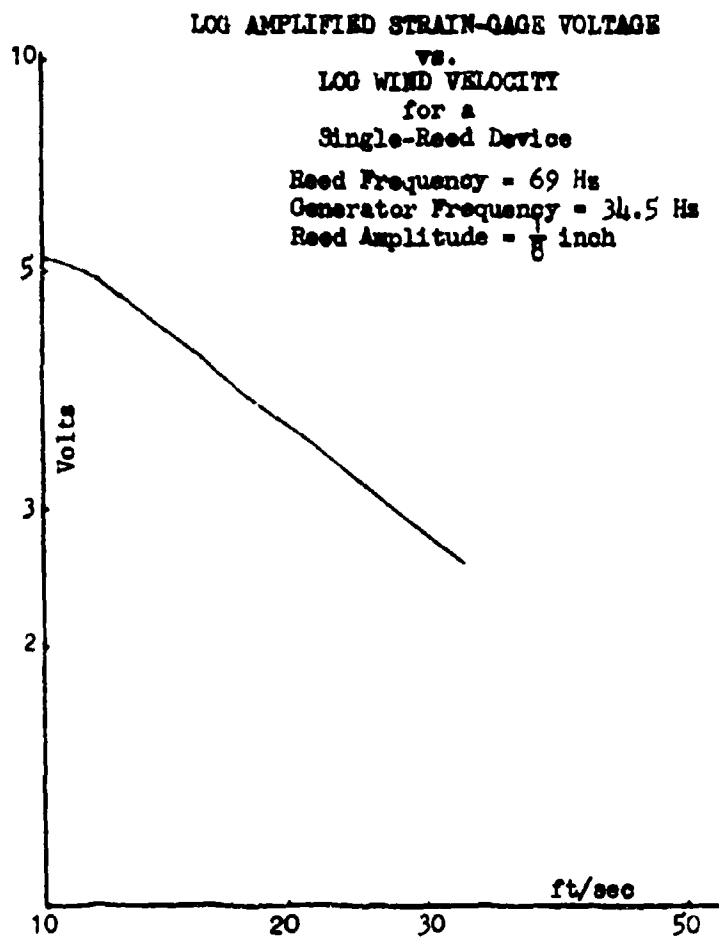Generator Frequency = 34.5 Hz
Reed Amplitude = $\frac{1}{8}$ inch

FIGURE 7.   Constant-Driver-Coil-Current Plot.

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Generator Voltage (Peak to Peak)
vs
Alignment Error
(Dual Reeds Vibrating at Constant Amplitude)
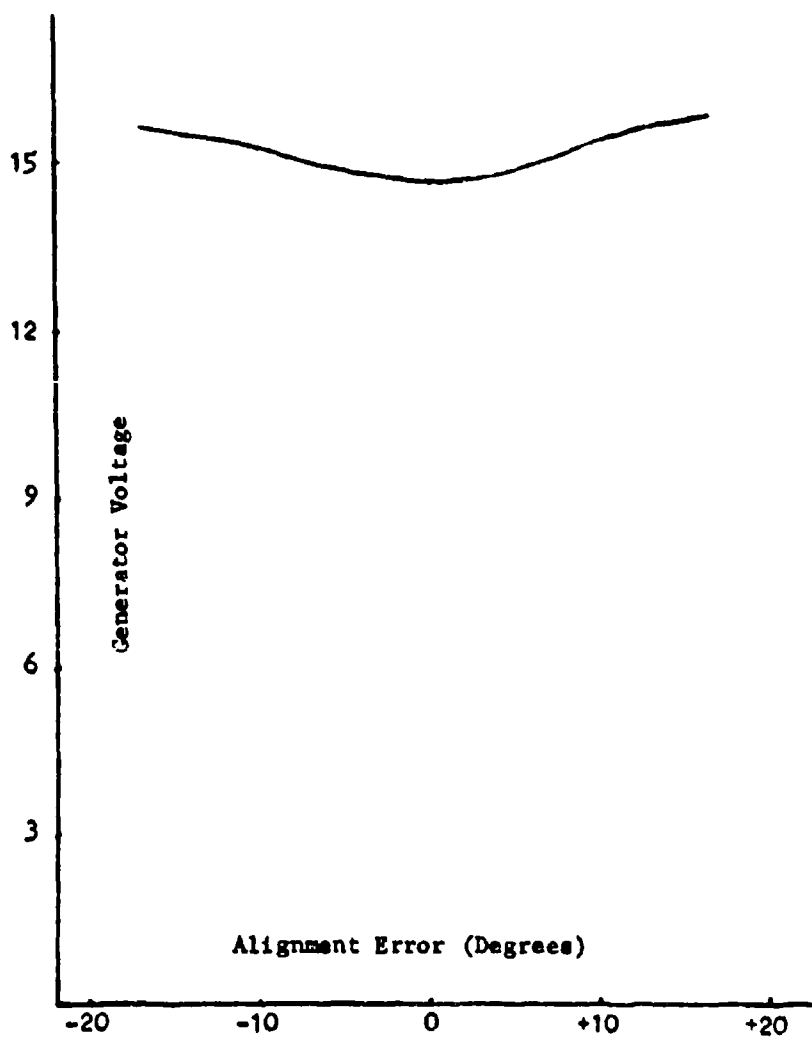
Wind Velocity =   20 ft/sec

FIGURE 8.   Effect of Alignment Error.

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to drive only one reed. But, in actuality, for equal oscillation amplitudes, the slopes are found to be almost equal. The reason for this apparent inconsistancy is that the minute power expended in overcoming the aerodynamic lift forces is much less than that consumed by mechanical, magnetic and copper losses. A rather involved calculation (not shown) using the classical value of 2π for k reveals that for a reed oscillation amplitude of one-eight inch and wind velocity of 20 ft/sec, the average power expended on lift forces is approximately 10% of the driver-coil input power. The proportionality of driver-coil current to lift force, as expressed in subsection II.A, is a much more practical way to view the operation of the device.

Figure 7 is a log-log plot obtained using the constant-driver-coil-current approach of figure 5. If the curve were extended to the left, it would bend down and approach a horizonal asymptote. It may seem a bit surprising that the curve is almost linear in spite of the high mechanical losses. However, the mechanical losses are very nearly directly proportional to the amplitude of reed oscillations, so they do not destroy the linearity.

Figure 8 is a constant-reed-amplitude plot showing the effect of a deliberate alignment error. As predicted in subsection II.D, the alignment is not of critical importance.

All of the curves shown have been "smoothed". The measurement errors were many and often compounded. Moreover, the effect of wind-tunnel blockage was considerable. In the subsection of section V entitled COEFFICIENT OF LIFT, the suggestion is made that perhaps a slightly concave line should have been used as the best fit line for the upper portion of the curves in figure 6.

V.   CONCLUSIONS AND SUGGESTIONS

The wind-tunnel data presented in the previous section tend to affirm the validity of the many assumptions and approximations that were made in the development of the theory presented in the THEORY OF OPERATION section. Regrettably, a variable-density wind tunnel was not available, so the effect of varying p could not be experime tally observed. However, since the major approximations have been shown to be essentially valid, there is little reason to doubt that p and v are approximately inversely proportional over the linear region in which $v > 3V_{po}$. For the remainder of this paper, it will be assumed that that inverse relationship exists. However, see the COEFFICIENT OF LIFT subsection where this relationship is questioned.

The analysis till now has been primarily limited to first-order effects. For example, no attempt has been made to investigate in detail such secondary effects as:

1. Aeroelasticity effects which occur due to the turbulance of the fluid.

2. Changes in the magnetic coupling of the reed to the driver due to the displacement of the reed's average position by the d.c. component of the magnetic driver force and/or alignment error.

3. Deviations from a linear magnetic-flux-verus-coil-current relationship (hysteresis).

4. Operation in the $v < 3V_{po}$ region.

5. Source and magnitude of mechanical and magnetic losses.

6. Effect of temperature changes on reed elasticity, frequency, and magnetic losses.

7. Wind-tunnel blockage.

The specific purpose of this paper is to propose a vibrating-reed pv (or mass-rate-of-flow) meter based on the theory in section II and to show from wind-tunnel data on a laboratory model that the pv effect is present and of usable magnitude. However, some of the secondary effects mentioned above can be of serious consequence, affectir both the basic operation and accuracy of the device. In the remainder of this section,

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the major design considerations and obstacles likely to be encountered
are described in general terms. Where appropriate, practical suggestions
are offered which should greatly aid anyone wishing to duplicate these
efforts or produce a commercial pv instrument utilizing the vibrating-
reed concept. Possible applications and automatic operation schemes
are also examined.

A. REDUCTION OF MECHANICAL LOSSES

The proposed pv meter is by necessity of the "calibrated" type.
Since the calibration depends on the mechanical losses of the system,
every effort should be made to hold these losses constant. This is
in general a difficult task to achieve if a single-reed device is to
be relocated after calibration, because the mechanical Q of the reed
and its mount depend to a very large extent on the amount of energy
that is absorbed by the reed mount and its surroundings. Early in
the initial experiments, it was discovered that the mechanical losses
for a single-reed device varied by a factor of as much as five whenever
the device was moved from one locale to another. The operation of
a properly tuned dual-reed device does not exhibit such dependence on
the surroundings. The mechanical isolation principle involved in the
dual-reed device is that used in the common tuning fork. Two matched
reeds are mounted in parallel and are driven in opposite directions.
Although both reeds are constantly in motion, their total momentum
at every instant of time is essentially zero. Thus very little of
the meter's vibrational energy is transmitted to the surroundings
via the base. In other words, the base of a tuning-fork arrangement
does not undergo significant vibration, and thus can transmit no
where near the energy out of the system that the base of a single-
reed device normally does. The major advantage of the dual-reed sys-
tem is that the mechanical losses are much easier to hold constant.

The mechanical losses in a single-reed system can be significantly
reduced in two different ways. The first is to bolt the reed mount to
a heavy elastic base; preferably one whose natural resonant frequency
matches that of the reed. In this type of installation the base acts
as a sounding board and reflects the transmitted energy back to the
reed in the form of standing waves.

The second method is to reduce the mechanical coupling between the reed mount and its surroundings. This approach was used to generate the data for the single-reed plots of figures 6 and 7. The device was suspended from the top of the wind tunnel. With the device operated in this "upside down" fashion, the mechanical losses proved to be considerably lower than those encountered in the "right side up" position.

Throughout the entire series of experiments, the mechanical losses proved to be the variables most critical and most difficult to control. Any practical design of a vibrating-reed pv meter should integrate the mechanical properties of reed, mount, and base with those of the surroundings (the conduit). It is strongly recommended that no one attempt the design of a commercial device without the aid of a good vibrations engineer.

B. MAGNETIC COUPLING OF REED TO DRIVER

Several different schemes could be used to drive the reed magnetically. The one used by the author had several merits worth mentioning (see figure 3). The pole pieces are streamlined and are inclined to the fluid flow in a manner that allows smooth flow over the reeds. The drive pole pieces are located far enough down the reeds so that the airflow over the reed tips, which undergo the most displacement and are most sensitive to aeroelasticity effects, is effectively removed from the turbulence produced by the driver pole pieces and coil. Each of the reed blades has attached to it a small pole piece of soft iron which fits inside the driver pole pieces. These bits greatly increase the magnetic coupling between reed and driver, thus allowing a considerable reduction in the driver current and/or number of coil turns required, and insure operation on the linear portion of the B-verus-H curve. In turn, the reduced wire size and/or fewer turns make possible reduced driver-coil dimensions, thus minimizing the effect of wind-tunnel blockage.

C. WIND-TUNNEL BLOCKAGE

Wind-tunnel blockage will of course be a primary consideration if a vibrating-reed device is to be installed inside a pipe to monitor the pv-product (or mass-rate-of-flow) of a moving fluid. Extreme care would have to be taken to ensure that the velocity of the fluid passing over the reed surface has the same proportion to the average

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velocity of the fluid in the conduit throughout the range of desired
operation. It should be mentioned that all of the velocity data
presented in section IV refers to the wind velocity near the reed and
not the average velocity through the wind tunnel. Use of the magnetic
coupling scheme discussed in subsection B plus the use of the smallest
wire size possible will do much toward reducing the blockage.

The ideal solution is to channel the fluid through a conduit which
has an elongated cross-section built around the reed and driver pole
piece. The driver coil could be positioned outside the conduit,
sealed off, and thus completely isolated from the fluid. The pole
piece could be horizonally streamlined and if placed near the reed
mount could make the blockage effect near the "working" part of the
reed negligible. Such an approach should make possible good accuracy
over a wide range of fluid density and velocity variations. Of course,
two channels would be required for a dual-reed device, one for each
reed with the magnetic driver coil located in between.

D. REED AMPLITUDE DETECTION

No matter which approach is used to obtain a pv-product from a
vibrating reed, some means for sensing the reed amplitude must be
employed. Many schemes for amplitude detection are possible for lab-
oratory models, including optical ones. For example, an aural detec-
tion method was utilized to collect the data for the plot of figure 6.
However, a commercial device would require a sensor that would yield
an electrical output. A strain gage bonded to the reed was used to
accomplish that function on the single-reed laboratory model because
of its implicity, small size, and good inearity. Its main disadvan-
tage was its low output - about two millivolts maximum when used with
a six-volt battery. Considerable amplication was required to obtain
good sensitivity on the voltage indicator. The battery voltage could
have been increased, but generous amplication would still have been
necessary. A much larger output voltage can be obtained from a moving
coil or phonograph pickup mounted onto the reed. A phonograph crystal-
type pickup could supply up to one volt. The optimum sensor to use
will probably depend upon the particular chemical enviroment.

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E. INDICATORS

In the constant-reed-amplitude approach, the magnetic-driver coil current (or coil voltage for a resistive driver) serves as a measure of the pv-product. It makes little difference if the average current, rms current, peak current or peak-to-peak current is used as the measure, so long as the shape of the waveform is not dependent on time. For a given waveform these quantities bare a constant relationship to each other. The only difference in effect will be the final scale factor, which is absorbed in the calibration process as is the B-verus-H relationship along with the mechanical, magnetic, and copper losses. It is not necessary that the waveform be sinusoidal. Nor is it required that the symmetry of the waveform be preserved under changes in the reed's dynamic loading - except in the case of mass metering - since the one-to-one relationship between current and average lift force is preserved. Analogous statements can be made for the driver coil voltage or the voltage across a resistor plus driver coil. For example, The peak-to-peak audio signal generator voltage (same as voltage across driver plus 30-ohm resister) was used to obtain the data for the curves in figure 6.

F. FACTORS LIMITING THE RANGE OF OPERATION

A vibrating reed can properly function as a pv-product indicator only so long as the value of the fluid velocity of v is greater than $3V_{po}$. Otherwise the device begins to operate in its non-linear region. The non-linear region is the "knee" of the plot in figure 6. The use of equation (2.14) leads to the expression

$$v > 3V_{po} \approx 3 \, w \, \text{Amplitude}$$

$$\triangleq \frac{3}{2\pi} \, \text{frequency Amplitude.} \qquad (5.1)$$

Thus it is easy to see that the lower limit of v below which the meter does not properly function is dependent on the product of reed frequency and amplitude, both of which are under the control of the designer.

The upper limit of v is determined primarily by two factors, the rigidity of the reed and the turbulence of the fluid. By making the reed stronger and less flexible, the effects of the d.c. component of the magnetic drive force and of the static lift force due to alignment error - both of which tend to distort the reed's average position thus changing the magnetic coupling - can be reduced. Of course, a heavier reed means an increase in mechanical losses, thus reducing the sensitivity for lower pv-products.

Perhaps the most severe limitation of the maximum pv-product which can be measured with any reliability is due to the effects of aeroelasticity or "buffeting" caused by fluid turbulence. The effect of buffeting closely resembles that of a time-varying alignment error. But since the pv^2 lift-force component due to buffeting is not constant, its effect is to set up superfluous oscillations in the reed whose phase sometimes aids and sometimes opposes the oscillations produced by the magnetic driver. The overall effect is a "mechanical noise" which modulates the amplitude of vibrations. Obviously, laminar flow is desired! A flow straightener immediately preceeding the laboratory model was found to be an absolute necessity. The author used a three-quarter-inch-thick piece of aircraft-wing honeycomb material to "straighted" the air flow in the wind tunnel. In this manner, satisfactory operation was easily obtained at speeds up to 28 ft/sec, the maximum velocity that the wind unnel could produce. No information is known as to the practical upper limit of v that the aeroelasticity effects will allow since it is also a function of reed dimensions, rigidity, amplitude of vibration, density of fluid, etc. Increasing the amplitude of vibrations will reduce the relative effect of buffeting but will also increase the lower limit of v and increase the mechanical losses.

G. MECHANICAL Q OF THE REED

The vibrating reed can be viewed as a low-frequency, high-Q resonant circuit. The driver produces oscillations by appling energy to the circuit at the reed's natural frequency. The reed's mechanical losses together with the absorbtion of its linear momentum by the fluid have the same dampening effect that the resistor in an ordinary RLC tank circuit provides. The same differential equations which describe the parallel RLC tank circuit can be used to describe the motion of the reed. However, the Q of a metal reed is much higher than that normally found in an electrical circuit.

At this point, the author will define the dynamic Q of the reed to be the ratio of the optimum driver-current frequency to the band-width separating the current frequencies for which the reed vibrates with an amplitude of 0.707 that exhibited at the optimum current frequency. The 0.707 amplitude is that for which the lift-force power drops by a factor of one-half.

The dynamic Q for the laboratory model was about 150 i- still air. At wind-tunnel speeds of 25 ft/sec the dynamic Q dropped to about 60. Note that the ratio of the dynamic Q's for zero ft/sec and 25 ft/sec is the inverse of the driver powers required to maintain constant reed amplitude at those speeds - see figure 6. In fact, a little reflection on the theory of operation reveals that it can be modified to show that the dynamic Q can also be used as a measure of the pv-product. This approach was not pursued because the dynamic Q is a much more difficult parameter to measure than is the driver current or driver voltage or reed amplitude.

The high mechanical Q could easily be the major objection to the use of a vibrating-reed, pv-product indicator. For example, if the driver power is supplied by an audio signal generator, the generator's frequency must be maintained to within a fraction of a Hertz if an accurate indication of the pv-product is to be obtained. One possible means of avoiding the expense of a frequency-stablized audio generator is to tune the reeds to 120Hz. Then a standard 60-Hz power outlet could be substituted for the generator. Another, more practical solution, is to provide the driver power from an oscillator built

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around the reed itself. A simple scheme for doing this is presented in figure 9 below.
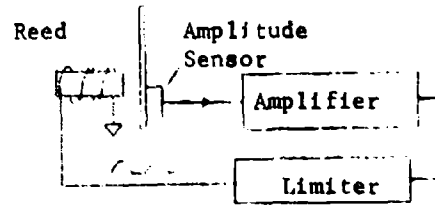

FIGURE 9. Oscillator Built Around The Reed.

A limiter or frequency divider is necessary in the oscillator loop because of the driver current frequency must be one-half that of the reed, as shown by equation (2.11).

## H. AUTOMATIC MASS-RATE-OF-FLOW METERING

Figure 9 describes a method for driving the reed at its natural frequency by using the reed as the tank circuit of an oscillator. Since both the driver current and the amplitude of vibration will change with the aerodynamic loading on the reed, some means of controlling the amplifier gain must be employed if either the constant-reed-amplitude or constant-driver-current approach is to be used. Figure 10 contains two schematic diagrams illustrating how feedback might be used to control the amplifier gain in a manner that would yield the two approaches. The diode shown in parrallel with the driver coil functions as the limiter in figure 9.

To adapt either of the two systems to a constant-mass-rate-of-flow situation, the indicator input voltage could be matched with a reference voltage to produce an error signal that would control a valve upstream of the reed. Thus the pv-product (or mass-rate-of-flow) for a fluid moving through a conduit could be held constant over a wide range of fluid densities and velocities.

Another advantage of feedback would be the reduction of the device's transit time. Without feedback, one to two seconds - depending on the aerodynamic loading on the reed - are required for the amplitude of vibrations to approach steady state after a change in driver current amplitude. Feedback could significantly reduce that inherent delay.
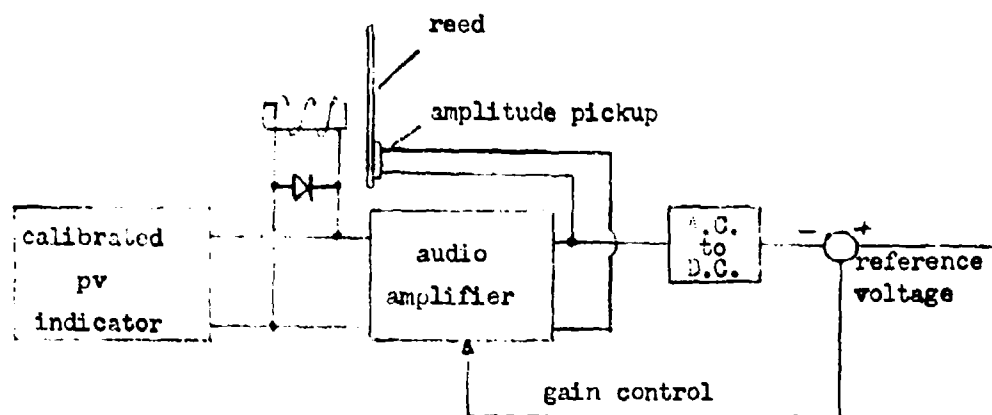
A. Constant-Reed-Amplitude Approach
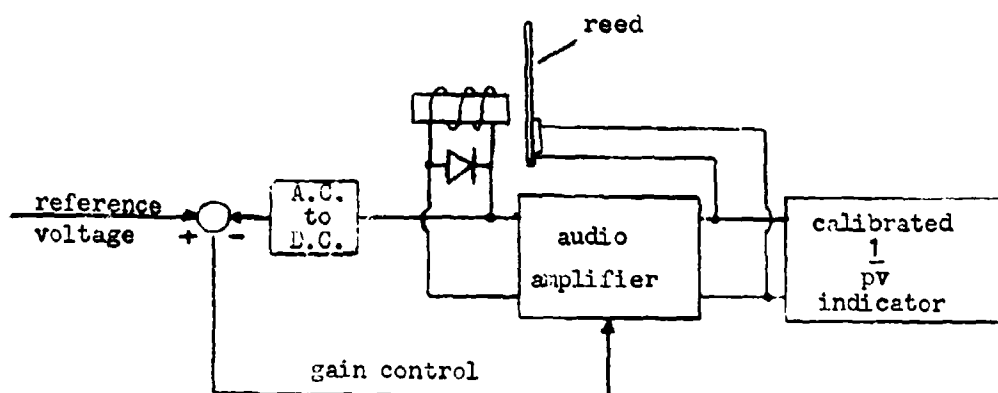
B. Constant-Driver-Current Approach

FIGURE 10   Schematic Diagrams for Automatic pv Metering.

I. MASS METERING WITH A VIBRATING REED

With some additional circuitry, the constant-reed-amplitude can be extended to provide mass metering. The method is outlined below.

First, to obtain mass from a pv-product, it is necessary to multiply the pv-product of the fluid by the effective area of the conduit (in which pv is measured), then integrate with respect to time as expressed in equation (5.2) below.

$$m(T) = \int_0^T p(t)v(t) \quad \text{conduit area} \quad dt \qquad (5.2)$$

Secondly, referring to one of the plots in figure 6, it is readily seen that the linear portion of the curve can be mathematically expressed as

$$V_g^2 = k\,p\,v + V_{gL}^2 \qquad (5.3)$$

where:

V_g = signal generator voltage.

kp = slope of linear portion of the curve.

V_{gL}^2 = square of voltage required to overcome the mechanical losses (point C or D in figure 6).

Solving equation (5.3) for pv yields

$$pv = \frac{V_g^2 - V_{gL}^2}{k} \qquad (5.4)$$

Substitution of equation (5.4) into equation (5.2) gives

$$m(T) = \int_0^T \frac{V_g^2(t) - V_{gL}^2}{k} \quad \text{(conduit area)} \quad dt. \qquad (5.5)$$

Therefore, an indication of the fluid mass passing the reed during a time interval T can be obtained by squaring the driver current (or voltage for a resistive driver), subtracting from the average a constant portion representing the mechanical losses, and integrating the result.

The squaring, subtracting and integrating process can be done in a variety of ways. Two possible approaches are listed below.

1. Square with a Hall-effect device, subtract from the average output voltage a d.c. voltage representing the mechanical losses, and integrate electronically with a pulse generator and counter.

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2. Use an a.c. watt-hour-type meter that has two opposing torque
producers geared to the same shaft. The driver voltage and current
would be the primary torque-generator inputs. Regular 60 Hz
current and voltage applied to the oposing torque generator could
represent the mechanical losses.

J. VIBRATING REED AS A FLUID-DENSITY METER.

The vibrating reed device can also be used as a fluid-density
meter. The basic argument is as follows.

For a reed vibrating inside of a fluid-filled closed container,
the aerodynamic force on the reed can be approximated by

$$\text{Drag Force} = \tfrac{1}{2} \, C_D \, p \, v_p^2 \qquad (5.6)$$

where C_D is defined to be the complex drag coefficient for a flat
plate vibrating inside an enclosure. The use of the same approximations
used in section II yields

$$i_d = \text{function of } p \text{ and } V_{po}^2. \qquad (5.7)$$

Since V_{po} is directly proportional to the amplitude of vibrations, then

$$i_d = \text{function of } p \text{ and } (\text{Amplitude})^2. \qquad (5.8)$$

The exact relationship will depend on the driver coil, copper losses,
magnetic losses, mechanical losses, reed dimensions, frequency of
vibration, viscosity of the fluid, etc. But for any given fluid,
reed device, and container, a one-to-one correspondence between current
and density exists which can be determined by calibration. Either
the constant-driver-current or constant-reed-amplitude approach could
be used. The accuracy obtained will be largely dependent on how small
the mechanical losses can be made. The copper and magnetic losses are
a function of the driver-coil current. Their effect will be to change
the scale factor, but they should not significantly affect the basic
accuracy of the meter. No laboratory investigations were performed
to test the above argument, but the plots in figure 6 show that for
the author's laboratory model the power required to "drag" the reed
through still air is roughly equal to that expended in overcoming the
mechanical losses.

K. COEFFICIENT OF LIFT

One assumption was made in section II which was probably not
justified. In the transition from equation (2.5) to equation (2.7),
k' was assumed to remain constant. This may have been a rather gross
approximation. In section 6.9 of his book, The Theory of Aeroelasticity,
Y. C. Fung examines the circulatory lift on a flat plate undergoing
forced oscillations in a windstream. The geometry is remarkably
similar to that of the vibrating reed. In his analysis, Fung shows
that the coefficient of lift for a plate undergoing sinusoidal oscil-
lations is not only complex, but also a function of frequency, fluid
velocity, plate width, and angle of attack.

If Fung's results are applied to the reeds of the test model, it is
found that the linear portions of the plots in figure 6 should be
slightly concave. In other words, the slope should increase with the
fluid velocity. In particular, the slope at 25 ft/sec should be
about 14 percent greater than that at 10 ft/sec. The concave nature
of the curves was not detected in the experimental testing, but it
could easily have been concealed by wind-tunnel blockage and
aeroelasticity effects.

If, in fact, the current-squared-verus-velocity curves are
slightly concave, the value of the vibrating reed as a mass-rate-of-
flow indicator need not be seriously degraded. Proper design of the
reed's dimensions, frequency, and relative placement in the wind-
stream together with control of the effective conduit cross-section
and the wind-tunnel blockage effect could do much toward linearizing
the curves. I⁺ is the integral of pv over the entire conduit cross-
section that is important in mass-rate-of-flow metering - not just
the pv-product of the fluid in the immediate vicinity of the reed.
Since the coefficient of lift is not dependent on p, a constant slope
for the current squared versus average fluid velocity curve (constant-
reed-amplitude) would mean true mass-rate-of-flow metering. Obviously,
before an accurate meter can be made, considerable research must be
done in this area by someone with an aeronautical engineering back-
ground.

VI. <u>SUMMARY</u>

It has been shown that the vibrating reed can be used to measure
the density of a static fluid or the density-velocity product of a
moving fluid. It is readily adaptable to mass-metering situations and
can be used to automatically control a valve that would maintain the
pv-product of the fluid moving through it within loosely specified limits,
over a wide range of densities and velocities. Furthermore, the device is
fairly rugged, inexpensive and can be made to withstand a corrosive chem-
ical enviroment.

The engineering efforts needed for commercial manufacture of the
device are expected to be straightforward and should not require any
state-of-the-art techniques. Although many design problems will have
to be solved, the author has already outlined approaches that could be
used to resolve the major difficulties that are likely to be encountered.

Regrettably, no statement can be made yet as to the ultimate accuracy
that a vibrating-reed instrument could provide. Overall accuracy is
expected to be rather low as compared to that of some other fluid-measuring
instruments; however, the convenient electrical outputs and ease of
adaptation to automatic-control situations exhibited by the vibrating-reed
meter are highly desirable and certainly justify further investigation.

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13 ABSTRACT

For many fluid-mass-rate-of-flow metering situations, a measure of the fluid's density-velocity product is required. The density-velocity (ρv) product is multiplied by an effective conduit cross-sectional area to yield the mass-rate-of-flow. The area multiplication is accomplished by simply changing the scale of the ρv-product indicator.

The purpose of this paper is to show how a magnetically-driven vibrating reed can be used to measure either the ρv product of a fluid or its mass-rate-of-flow through a conduit. The proposed meter differs from the rotating-vane mass-rate-of-flow meters in that it operates on a transverse rather than angular momentum exchange.

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Mass Flow Meter						
Mass Metering						
Fluid Flow Metering						
Flow Meter						

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